Higher Dimensional Gravity and Local AdS Symmetry

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Requiring general covariance and second order field equations for the metric implies that gravitation in higher dimensions can be described by theories with higher powers in the curvature. The most general theory of this kind in d dimensions has $[(d-1)/2]$ free parameters. It is shown that by allowing the existence of a sector with non-vanishing torsion in the theory, these parameters become fixed in terms of the gravitational and the cosmological constants. In even dimensions, the Lagrangian is written as a Born-Infeld-like theory. In odd dimensions, the Lagrangian is a Chern-Simons form for the (A)dS or Poincaré local symmetry groups. Consistency of equations of motion implies that torsion may occur explicitly in the Lagrangian only for $d = 4k - 1$. These torsional Lagrangians are related to the Chern characters in $4k$ dimensions. The coefficients of the different terms in these Lagrangians can be shown to be quantized. These theories posses a large class of interesting solutions, including black holes and homogeneous cosmologies.

I. INTRODUCTION

The possibility that the spacetime dimension be greater than four has been explored in the context of unified quantum field theories, ranging from Kaluza-Klein (super)gravity to superstrings and M-theory. Although many different approaches have been followed in the generalizations to $d > 4$, little attention has been given to purely geometrical higher-dimensional theories of gravity. As a consequence, most models have been conceived within the simplest generalization of General Relativity to higher dimensions.

A. Beyond the Einstein-Hilbert action

It is the standard practice to describe the gravitational field assuming the spacetime geometry as given by the Einstein-Hilbert (**EH**) action –with or without cosmological constant. This theory is the most reasonable choice in dimensions three and four[1], but it is not necessarily so for $d > 4$. Indeed, we would like to discuss some attractive alternatives that exist in higher dimensions when the EH assumption is relaxed.

The idea that a more general theory could be employed to describe the spacetime geometry in dimensions larger than four –even in the absence of torsion– was first considered some sixty years ago by Lanczos [3]. More recently, it was observed that the low energy effective Lagrangian for gravity obtained from string theory would have curvature-squared terms [4], which are a potential source of inconsistencies as they would in general bring in ghosts. However, it was soon pointed out by Zwiebach [5] and Zumino [6], that if the effective Lagrangian would contain the higher powers of curvature in particular combinations, only second order field equations are produced and consequently no ghosts. The effective Lagrangian obtained by this argument, was precisely of the form proposed by Lanczos for $d = 5$ and, for general d, by Lovelock [7].

On the other hand, in the more recent context of M-theory there are further clues that point in this same direction. For instance, it is expected that the low energy regime of M-theory should be described by an eleven dimensional supergravity of new type with off-shell local supersymmetry [8]. Moreover, the perturbation expansion for graviton scattering has led to conjecture that the new supergravity Lagrangian should contain higher powers of curvature [9]. A family of supergravity theories that satisfy these conditions has been recently proposed [10,11], and it should be expected that the purely gravitational sector of those new theories to be an extension of ordinary EH gravity, as described below.

The previous discussion underscores the need to address the question about the minimal requirements for a consistent theory which includes gravity in any dimension. Those requirements should include both general covariance and second order field equations for the metric. For $d > 4$ the most general action for the metric satisfying these criteria is a polynomial of degree $[d/2]$ in the curvature[2], the Lanczos-Lovelock **(LL)** theory.

[1] In 1+1 dimensions, in order to write an action principle it is necessary supply the theory with an extra scalar field [1,2].

[2] Here $[x]$ represents the integer part of x.

B. First order formalism

In standard General Relativity, the affine structure of spacetime (connection) is assumed to be derived from the metric. The link between the two structures is the torsion equation, which is often assumed as an off-shell requirement for the spin connection. This assumption implies that there are no independent propagating degrees of freedom for torsion. However, in a theory containing fundamental spinors coupled to gravity, it is necessary that the metric and affine properties of spacetime be treated separately. A purely metric formulation, would be sufficient for the description of spinless particles and fields because they only couple to the symmetric part of the affine connection. On the other hand, spinors provide a basis of irreducible representations for $SO(d-1,1)$, or $SO(d-1,2)$, but not for $GL(d)$. Thus, spinors are naturally defined relative to a local frame on the tangent space rather than in relation to a coordinate system on the base manifold. Furthermore, spinors couple to the spacetime torsion through minimal coupling with the spin connection (ω_μ^{ab}), while the coupling with the spacetime metric follows from a completely different relation with the vielbein (e_μ^a).

In a theory containing fermions, it is therefore more natural to look for a formulation of gravity in which ω^{ab} and e^a are dynamically independent, with curvature and torsion standing on similar footing. Thus, the first order formalism should be assumed allowing the theory to determine whether the spin connection is an independently propagating field or not.

In three and four dimensions, allowing ω and e to be dynamically independent does not modify the standard picture in practice because any occurrence of torsion in the action leads to torsion-free classical solutions (in vacuum)[3]. In higher dimensions, however, theories that include torsion can be dynamically quite different from their torsion-free counterparts [12].

As we shall see below, the dynamical independence of ω^{ab} and e^a also implies that the gravitation theories in $d = 2n-1$ can be defined on a fiber bundle, like a Yang-Mills theory, a feature that is not shared by General Relativity except in three dimensions.

II. HIGHER DIMENSIONAL GRAVITY: LANCZOS-LOVELOCK THEORY

The LL theory regards gravity as the result of local deformations of a d-dimensional Riemannian manifold. Its Lagrangian can be defined in four independent ways:

[3]In the case of coupling to spinning matter, the torsion equations allow expressing ω in terms of e and the matter fields.

(**i**) As the most general invariant constructed from the metric and curvature leading to second order field equations for the metric [3,7].

(**ii**) As the most general d-form invariant under local Lorentz transformations, constructed with the vielbein, the spin connection, and their exterior derivatives, without using the Hogde dual $(*)$ [13].

(**iii**) As a linear combination of the dimensional continuation of all the Euler densities of dimension $2p < d$. [6,14].

(**iv**) As the most general low energy effective gravitational theory that can be obtained from string theory [5].

Definition (**i**) was historically the first. It is appropriate for the metric formulation and assumes vanishing torsion. Definition (**ii**) is slightly more general than (**i**) and allows for a local first-order formulation, and for torsion-dependent terms in the action [12]. As a consequence of (**ii**), the field configurations that extremize the action obey first order equations for ω and e.

Assertion (**iii**) gives directly the LL Lagrangian as a polynomial of degree $[d/2]$ in the curvature,

$$I_G = \int \sum_{p=0}^{[d/2]} \alpha_p L^{(p)}, \tag{1}$$

where α_p are arbitrary constants, and

$$L^{(p)} = \epsilon_{a_1 \cdots a_d} R^{a_1 a_2} \cdots R^{a_{2p-1} a_{2p}} e^{a_{2p+1}} \cdots e^{a_d}, \tag{2}$$

(wedge product of forms is understood throughout).

Statement (**iv**) reflects the empirical observation that the vanishing of the superstring β-function in $d = 10$ gives rise, in the low energy limit, to an effective action of the form (1) [5]. In even dimensions, the last term in the sum is the Euler density, which does not contribute to the equations of motion (although in the quantum theory, this term in the partition function would assign different weights to nonhomeomorphic geometries).

Note that the first two terms in the LL action (1) are the EH theory. Although General Relativity is contained in the LL action as a particular case, the actions with higher powers of curvature define theories which are dynamically very different from EH, and the classical solutions of the full action are not even perturbatively related to those of Einstein's theory. However, to lowest order in perturbation theory around a flat, torsion-free background, all of the $L^{(p)}$ defined in (2) with $p > 2$ are total derivatives [6].

The $[(d+1)/2]$ dimensionful constants α_p in the LL action contrast with the two constants of the EH theory (G and Λ). This feature could seem as an indication that renormalizability of the LL theory would be even more unattainable than in ordinary gravity. However, this is not necessarily so. As is shown below, there are some very special choices of α_p such that in odd dimensional spacetimes the theory becomes invariant under a larger gauge group. In this case the values of the coupling constants α_p are protected by gauge invariance, which, if not spoiled by anomalies, should actually improve renormalizability [15,16]. In the remaining of this section the α_p's are selected according to the criterion that the integrability (or consistency) conditions for the field equations would not impose additional algebraic constraints on the curvature and torsion tensors.

A. Consistency of Equations of Motion

Consider the action (1), viewed as a functional of the spin connection and the vielbein, $I_G = I_G[\omega^{ab}, e^a]$. Varying with respect to these fields, the generalized Einstein equations are obtained,

$$\delta e^a \to \mathcal{E}_a = 0, \tag{3}$$
$$\delta \omega^{ab} \to \mathcal{E}_{ab} = 0, \tag{4}$$

where we have defined

$$\mathcal{E}_a := \sum_{p=0}^{[\frac{d-1}{2}]} \alpha_p (d-2p) \mathcal{E}_a^p, \tag{5}$$

$$\mathcal{E}_{ab} := \sum_{p=1}^{[\frac{d-1}{2}]} \alpha_p p (d-2p) \mathcal{E}_{ab}^p, \tag{6}$$

and

$$\mathcal{E}_a^p := \epsilon_{ab_1\cdots b_{d-1}} R^{b_1b_2} \cdots R^{b_{2p-1}b_{2p}} e^{b_{2p+1}} \cdots e^{b_{d-1}}, \tag{7}$$
$$\mathcal{E}_{ab}^p := \epsilon_{aba_3\cdots a_d} R^{a_3a_4} \cdots R^{a_{2p-1}a_{2p}} T^{a_{2p+1}} e^{a_{2p+2}} \cdots e^{a_d}. \tag{8}$$

Equations (3) and (4) are independent Lorentz tensor $(d-1)$-forms, which should be solved for ω^{ab} and e^a. By virtue of the Bianchi identity, it is easy to check that the exterior covariant derivative of (7) is related to (8) as

$$D\mathcal{E}_a^p = (d-2p-1) e^b \mathcal{E}_{ba}^{p+1}, \tag{9}$$

where $0 \leq p \leq [(d-1)/2]$. This leads to the condition

$$D\mathcal{E}_a = \sum_{p=1}^{[\frac{d+1}{2}]} \alpha_{p-1}(d-2p+2)(d-2p+1) e^b \mathcal{E}_{ba}^p = 0. \tag{10}$$

On the other hand, taking the exterior product of (6) with e^b gives

$$e^b \mathcal{E}_{ba} = \sum_{p=1}^{[\frac{d-1}{2}]} \alpha_p p (d-2p) e^b \mathcal{E}_{ba}^p = 0. \tag{11}$$

Comparing the last two equations, one can see that they are consistent for a generic choice of the coefficients α_p only if $e^b \mathcal{E}_{ba} = 0$ for all p. This in turn would imply that for a generic configuration torsion must vanish.

An interesting alternative way to curb the proliferation of arbitrary coefficients in the action is to consider the case in which the first order formalism is strictly necessary, that is, when T^a is not forced to vanish identically. Having this in mind, the strategy will be to exploit the consistency of the field equations in order to obtain relations that fix the α_p's. It is at this point that even and odd dimensions differ radically: equations (10) and (11) have the same number of terms for $d = 2n-1$ but not for $d = 2n$. We will treat each case separately.

1. $d = 2n$: Born-Infeld-Like Gravity

Obviously $T^a = 0$ solves (4). However, this equation does not necessarily imply the vanishing of torsion in general. First, one can observe that (4) is an exterior covariant derivative,

$$\mathcal{E}_{ab} = D\mathcal{T}_{ab} = 0, \tag{12}$$

where we have defined the $(d-2)$-form,

$$\mathcal{T}_{ab} := \sum_{p=1}^{[\frac{d-1}{2}]} \alpha_p p \mathcal{T}_{ab}^p, \tag{13}$$

with

$$\mathcal{T}_{ab}^p = \epsilon_{aba_3\cdots a_d} R^{a_3a_4} \cdots R^{a_{2p-1}a_{2p}} e^{a_{2p+1}} \cdots e^{a_d}. \tag{14}$$

Note that $\mathcal{T}_{ab}^p$ satisfies the relations

$$e^b \mathcal{T}_{ab}^p = \mathcal{E}_a^{p-1}, \tag{15}$$

$$D\mathcal{T}_{ab}^p = (d-2p)\mathcal{E}_{ab}^p, \tag{16}$$

for $1 \leq p \leq [\frac{d-1}{2}]$. Differentiating both sides of (15) and using (16) one obtains that for $d = 2n$, equation (10) can also be written as

$$D\mathcal{E}_a = \sum_{p=1}^{n} \alpha_{p-1}(d-2p+2)[T^b \mathcal{T}_{ab}^p - (d-2p) e^b \mathcal{E}_{ba}^p] = 0. \tag{17}$$

Comparing with (6) and (13), one can see that (17) is satisfied without requiring vanishing torsion only if the coefficients α_p satisfy the following recursion relation:

$$\lambda(d-2p+2)\alpha_{p-1} = p\alpha_p, \tag{18}$$

for some real number λ. Then, equation (17) reads,

$$D\mathcal{E}_a = \frac{1}{\lambda}(T^b \mathcal{T}_{ab} - e^b \mathcal{E}_{ab}) = 0. \tag{19}$$

Therefore, in this case, the space of solutions of the equations of motion does not require the vanishing of torsion nor $\mathcal{T}_{ab}$, but only that T^a be a null vector of $\mathcal{T}_{ab}$.

The solution of equation (18) is given by[4]

$$\alpha_p = \alpha_0 (2\lambda)^p \left(\begin{array}{c} n \\ p \end{array} \right), \tag{20}$$

with $0 \leq p \leq n-1$. In this case we have only two independent fundamental coefficients in the Lagrangian, namely α_0 and λ, which are related to the d-dimensional Newton's constant (κ) and the cosmological constant (Λ) through[5]

$$\begin{aligned} \alpha_0 &= \frac{\kappa}{dl^d}, \\ \lambda &= -sgn(\Lambda)\frac{l^2}{2}. \end{aligned} \tag{21}$$

This choice of coefficients allows writing the equations of motion in a simpler form using the combination $\bar{R}^{ab} = R^{ab} + \frac{1}{l^2}e^a e^b$, in fact, (3) and (4) can be written as

$$\begin{aligned} \delta e^a &\rightarrow \epsilon_{ab_1\cdots b_{d-1}} \bar{R}^{b_1 b_2} \cdots \bar{R}^{b_{d-3}b_{d-2}} e^{b_{d-1}} = 0, \\ \delta\omega^{ab} &\rightarrow \epsilon_{aba_3\cdots a_d} \bar{R}^{a_3 a_4} \cdots \bar{R}^{a_{d-3}a_{d-2}} T^{a_{d-1}} e^{a_d} = 0. \end{aligned} \tag{22}$$

In particular, one could consider a sector for which

$$\mathcal{T}_{ab} = \frac{\kappa}{2}\epsilon_{aba_3\cdots a_d} \bar{R}^{a_3 a_4} \cdots \bar{R}^{a_{d-1}a_d} = 0, \tag{23}$$

which solves the equations of motion (22) identically without requiring $T^a = 0$. Another interesting consequence of the choice of coefficients (20) is that the Lagrangian takes the form

$$L = \frac{\kappa}{2n}\epsilon_{a_1\cdots a_d} \bar{R}^{a_1 a_2} \cdots \bar{R}^{a_{d-1}a_d}. \tag{24}$$

This is the pfaffian of the 2-form $\bar{R}^{ab}$, and in this sense it can be viewed as the Born-Infeld **(BI)**-like form [17],

$$L = 2^{n-1}(n-1)!\kappa\sqrt{\det\left(R^{ab} + \frac{1}{l^2}e^a e^b\right)}. \tag{25}$$

In four dimensions (25) reduces to a particular linear combination of the Einstein-Hilbert action with cosmological constant and the Euler density. Since the Euler density is a closed form, it does not contribute to the equations of motion. However, this term plays an important role in the definition of conserved charges for gravitation [18] as well as in the quantum theory.

The vanishing cosmological constant limit ($l \rightarrow \infty$) correspond to the Lagrangian

$$L_{(\Lambda=0)} = \frac{\kappa}{2}\epsilon_{a_1\cdots a_d} R^{a_1 a_2} \cdots R^{a_{2n-3}a_{2n-2}} e^{a_{2n-1}} e^{a_{2n}},$$

which is the dimensional continuation of the Gauss-Bonnet invariant in $(d-2)$ dimensions.

The two-form $\bar{R}^{ab}$ is a part of the AdS curvature (c.f. (29) below). This fact seems to suggest that the system might be naturally described in terms of an AdS connection [19]. However, that is incorrect: in even dimensions, the Lagrangian (24) is invariant under local Lorentz transformations and *not* under the entire AdS group. However, as shown in the next section, it is possible to construct gauge invariant theories of gravity under the full AdS group in odd dimensions.

2. $d = 2n-1$: *Local (A)dS Chern-Simons Gravity*

Odd-dimensional spacetimes are very special in that equations (10) and (11) have the same number of terms. Thus, if no further algebraic constraints on R^{ab} and T^a are assumed, equations (10) and (11) must be proportional ($e^b\mathcal{E}_{ba} = \lambda D\mathcal{E}_a$). This in turn is true if and only if the following recursion relation for the constants α_p is satisfied:

$$\lambda\frac{\alpha_p}{\alpha_{p+1}} = \frac{(p+1)(d-2p-2)}{(d-2p)(d-2p-1)} \tag{26}$$

where $0 \leq p \leq n-1$. Equation (3) has one more term than (4), but the exterior covariant derivative of the last term in (3) vanishes identically. The solution of (26) is given by

$$\alpha_p = \alpha_0 \frac{d(2\lambda)^p}{(d-2p)} \left(\begin{array}{c} n-1 \\ p \end{array} \right). \tag{27}$$

This particular choice of coefficients satisfies the consistency condition without requiring the vanishing of torsion or other subsidiary constraint.

On the other hand, equation (27) implies that in this case we have only two independent constants in the action, which can be expressed in terms of the gravitational and the cosmological constants in the same way as in even dimensions using (21). Choosing the coefficients α_p as in (27), implies that equations (3) and (4) are parts of a locally (A)dS covariant equation. This can be made explicit if ω and e are identified as different components of an (A)dS connection $\mathbf{A} = \frac{1}{2}\omega^{ab}J_{ab} + e^a J_{ad+1} = \frac{1}{2}W^{AB}J_{AB}$, where [6]

[4] The coefficient α_n –which multiplies the Euler density in (1)– cannot be fixed by the recursion relation (18) because this term does not contribute to the equations of motion. However, it is natural to choose $\alpha_n = \alpha_0(2\lambda)^n$, so that the formula (20) will also be valid for $p = n$.

[5] For any dimension, l is a length parameter related with the cosmological constant by $\Lambda = \pm\frac{(d-1)(d-2)}{2l^2}$. In the following we will choose the negative sign, but the analysis does not depend on this sign.

[6] Here, $\eta^{AB} = diag(-,+,\cdots,+,sgn(\Lambda))$. In this case, κ can be chosen proportional to l so that the action becomes dimensionless.

$$W^{AB} = \left[\begin{array}{cc} \omega^{ab} & e^a/l \\ -e^b/l & 0 \end{array} \right], \; A,B = 1,...d+1, \tag{28}$$

The corresponding AdS curvature is $\mathbf{F} = d\mathbf{A} + \mathbf{A}^2 = \frac{1}{2}\bar{R}^{AB}J_{AB}$, where

$$\begin{aligned} \bar{R}^{AB} &= dW^{AB} + W^A_C W^{CB}), \\ &= \left[\begin{array}{cc} R^{ab} + \frac{1}{l^2}e^a e^b & T^a/l \\ -T^b/l & 0 \end{array} \right], \end{aligned} \tag{29}$$

contains both the Riemann curvature and torsion tensors. Now, (3) and (4) can be written as different components of the equation

$$\delta W^{AB} \to \mathcal{E}_{AB} := \epsilon_{ABA_3 \cdots _{d+1}} \bar{R}^{A_3 A_4} \cdots \bar{R}^{A_d A_{d+1}} = 0. \tag{30}$$

This equation is manifestly covariant under local AdS symmetry generated by $\delta W^{AB} = -\nabla\lambda^{AB}$, where ∇ is the exterior covariant derivative in the AdS connection. More explicitly, the above statement implies that the theory is locally invariant under standard Lorentz symmetry ($\delta e^a = \lambda^a_{\ b} e^b$ and $\delta\omega^{ab} = -D\lambda^{ab}$), and also under local "translations",

$$\begin{aligned} \delta e^a &= -D\lambda^a \\ \delta\omega^{ab} &= \frac{1}{l^2}(\lambda^a e^b - \lambda^b e^a). \end{aligned} \tag{31}$$

The consequences of this extra symmetry will be discussed in section **IV.A**. It is simple to check that the condition $e^b \mathcal{E}_{ba} = \lambda D\mathcal{E}_a$ is automatically satisfied by virtue of the identity

$$\nabla \mathcal{E}_{AB} \equiv 0, \tag{32}$$

which is a consequence of the Bianchi identity, $\nabla \bar{R}^{AB} = 0$.

Consider now the action (1) for $d = 2n-1$ with the choice (27). The resulting Lagrangian is the Euler-Chern-Simons form, that is, its exterior derivative is the Euler form in $2n$ dimensions,

$$\begin{aligned} dL^{AdS}_{G\;2n-1} &= \frac{\kappa}{2n}\epsilon_{A_1 \cdots A_{2n}} \bar{R}^{A_1 A_2} \cdots \bar{R}^{A_{2n-1}A_{2n}} \\ &= \bar{\kappa}\mathcal{E}_{2n}. \end{aligned} \tag{33}$$

This action was proposed by Chamseddine [20] and also discussed in [21] in the context of torsion-free manifolds. In general, a Chern-Simons **(CS)** Lagrangian in $2n-1$ dimensions is defined by the condition that its exterior derivative be an invariant homogeneous polynomial of degree n in the curvature, that is, a characteristic class. In the case above, (33) defines the CS form for the Euler class $2n$ -form.

A generic CS Lagrangian in $2n-1$ dimensions for a Lie algebra g can be written as

$$dL^g_{2n-1} = \frac{2^{n-1}}{n}\kappa \left\langle \mathbf{F}^n \right\rangle, \tag{34}$$

where $\langle \; \rangle$ stands for a multilinear function in g, invariant under cyclic permutations such as the trace for an ordinary Lie algebra. In this case, the only nonvanishing brackets in the algebra are

$$\left\langle J_{A_1 A_2}, \cdots, J_{A_d A_{d+1}} \right\rangle = \epsilon_{A_1 \cdots A_{d+1}}. \tag{35}$$

Starting from the AdS theory (33) in odd dimensions, a Wigner-Inönü contraction obtained by taking the limit $l \to \infty$, yields a theory invariant under the Poincaré group. This same result is obtained by choosing $\alpha_p = \kappa\delta^{n-1}_p$ from the start. Then, the Lagrangian (1) becomes [7]

$$L^P_G = \kappa\epsilon_{a_1 \cdots a_d} R^{a_1 a_2} \cdots R^{a_{d-2}a_{d-1}} e^{a_d}. \tag{36}$$

In this way the local symmetry of (1) is extended from the Lorentz group ($SO(d-1,1)$) to the Poincaré group ($ISO(d-1,1)$). Analogously to the anti-de Sitter case, one can see that the action depends on the Poincaré connection $\mathbf{A} = e^a P_a + \frac{1}{2}\omega^{ab}J_{ab}$. It is straightforward to verify the invariance of the action (36) under local translations,

$$\delta e^a = -D\lambda^a, \;\; \delta\omega^{ab} = 0. \tag{37}$$

If λ is the Lie algebra-valued zero-form, $\lambda = \lambda^a P_a$, the transformations (37) are read from the general gauge transformation for the connection, $\delta\mathbf{A} = -\nabla\lambda$, where ∇ is the covariant derivative in the Poincaré connection. Moreover, the Lagrangian L^P_G is a Chern-Simons form. Indeed, with the curvature for the Poincaré algebra, $\mathbf{F}= d\mathbf{A} + \mathbf{A}^2 = \frac{1}{2}R^{ab}J_{ab} + T^a P_a$, the Lagrangian (36) satisfies (34), where the only nonvanishing components in the bracket are

$$\left\langle J_{a_1 a_2}, \cdots, J_{a_{d-2}a_{d-1}}, P_{a_d} \right\rangle = \epsilon_{a_1 \cdots a_d}. \tag{38}$$

B. Summary

The simplest example of Chern-Simons gravity occurs in $2+1$ dimensions, where the Einstein-Hilbert action with cosmological constant is a genuine *gauge* theory of the AdS group, while for vanishing cosmological constant it is invariant under *local* Poincaré transformations. Although this gauge invariance of $2+1$ gravity is not always emphasized, it lies at the heart of the proof of integrability of the theory [15].

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[7] This is the dimensional continuation of the Euler density from one dimension below, exactly as the three-dimensional EH Lagrangian without cosmological constant extends the two-dimensional Euler density. The local supersymmetric extension of (36) is also known [22].

As it has been shown in this section, there is only one theory of gravity of the LL family for each dimension which has a sector with unrestricted torsion. These theories fall into two families, namely BI for $d = 2n$, and CS for $d = 2n - 1$. For both families there are only two fundamental constants in the action, κ and Λ. In odd dimensions, κ is a dimensionless integer, and the action is invariant under local (A)dS transformation (or Poincaré when $\Lambda = 0$).

The theories described above (BI, CS, EH and their corresponding contractions) posses *a unique* maximally symmetric background solution (AdS or flat respectively). A more general family of theories with *unique* maximally symmetric background correspond to the choice:

$$\alpha_p = \frac{\alpha_0 d(2\lambda)^p}{(d-2p)} \left(\begin{array}{c} k \\ p \end{array} \right). \tag{39}$$

These theories reduce to EH for $k = 1$, and to BI and CS for $k = n - 1$, and can be shown to posses well behaved black hole solutions [23]. With the exception of $k = n - 1$ these theories must have vanishing torsion and will not be discussed in what follows.

C. Dynamical Behavior

When torsion is not set equal to zero, the standard variational principles –first, second and 1.5 order– are no longer necessarily equivalent. For example, varying the LL action with respect to e – in the "1.5 order formalism" [24], yields

$$\delta I_{LL} = \frac{\delta I_{LL}}{\delta e^a}\delta e^a + \frac{\delta I_{LL}}{\delta \omega^{bc}}\frac{\delta \omega^{bc}}{\delta e^a}\delta e^a,$$

which would reduce to the Einstein equations (3) (second order formalism) provided $\frac{\delta I_{LL}}{\delta \omega^{bc}} = 0$ (4). Thus, in particular, in the presence of spinors these formalisms would be different because in the second order case, the torsion equation (4) is imposed as a constraint, but this is in fact a matter of choice.

Another consequence of imposing a dynamical dependence between ω and e through the torsion equation is that it spoils the possibility of interpreting the local translational invariance as a gauge symmetry of the action. Consider the action of the Poincaré or (A)dS groups on the fields as given by (37) or by (31) respectively; taking the torsion condition $T^a = 0$ as a definition would imply

$$\delta \omega^{ab} = \frac{\delta \omega^{ab}}{\delta e^c}\delta e^c, \tag{40}$$

which would be inconsistent with the action of the local translations (37) on these fields. Thus, the spin connection and the vielbein –the soldering between the base manifold and the tangent space– cannot be identified as the compensating fields for local Lorentz rotations and translations –or (A)dS boosts–, respectively.

Thus, gravitation can be realized as a truly gauge theory with fiber structure, in which ω^{ab} and e^a are connection fields, only if the torsion is not constrained to vanish. As was shown above, this realization only exists for odd dimensions for the CS Lagrangian.

When torsion is set equal to zero, the number of degrees of freedom for a generic LL system has been shown to be the same as in the EH theory, namely $\frac{d(d-3)}{2}$ [14]. However, in the CS theory, without constraining the torsion, there are additional degrees of freedom [26] which can be associated to the contorsion tensor $k^{ab} := \omega^{ab} - \bar{\omega}^{ab}(e)$, where $\bar{\omega}^{ab}(e)$ is the solution of $T^a = 0$.

In view of the preceding analysis, there seems to be no reason to exclude torsion from the Lagrangian itself. In the next section we explore the possibility of adding torsion explicitly to the action.

III. ADDING TORSION IN THE LAGRANGIAN

The generalization of the Lovelock theory to include torsion explicitly is obtained assuming definition **(ii)**. This is a cumbersome problem due to the lack of an algorithm for classifying all possible invariants constructed from e^a, R^{ab} and T^a. In ref. [12] a useful recipe to generate all those invariants is given.

As with the LL Lagrangian, the explicit inclusion of torsion brings in a number of arbitrary dimensionful coefficients β_k analogous to the α_p's. Also in this case, one can chose the β's so as to enlarge the local Lorentz invariance into the AdS gauge symmetry[8] If no additional structure (e.g., inverse metric, Hodge dual $(*)$, etc.) is assumed, AdS invariants can only be produced in $4k$ and $4k - 1$ dimensions.

This can be seen as follows: invariance under AdS requires that the d-form be at least Lorentz invariant. Then, in order for these scalars to be invariant under AdS as well, it is sufficient that they be expressible in terms of the AdS connection (28). As is well-known (see, e.g., [27]), in even dimensions, the only d-form invariant under $SO(N)$ constructed according to the recipe mentioned above are the Euler character –for $N = d$ only–, and the Chern characters –for any N. An important difference between these invariants is that under a parity transformation the former is even while the latter is odd. Parity is defined by the sign change induced by a simultaneous inversion of one coordinate in the tangent

[8] Here $SO(d)$ and $SO(d-1,1)$ will be used indistinctly to represent the Lorentz group in d dimensions, while the d-dimensional (A)dS group will be denoted by $SO(d-1,2)$, $SO(d,1)$ or $SO(d+1)$.

space and in the base manifold. Thus, for instance the Euler characters, $\epsilon_{a_1\cdots a_d}R^{a_1a_2}\cdots R^{a_{d-1}a_d}$, are even under parity, while the Lorentz-Chern characters, $R^{a_1}_{\ b_2}\cdots R^{a_{2n}}_{\ a_1}$, and the torsional invariants such as $e_aR^a_{\ b}T^b$ are parity odd. In the previous section we discussed all possible Lagrangians of the form $\epsilon[R]^p[e]^{d-2p}$, in what follows we concentrate on the construction of the pure gravity sector as a gauge theory which is parity-odd. This construction was discussed in [29], and also briefly in [10,11].

In even dimensions the only AdS-invariant d-forms are, apart from the Euler classes, linear combinations of products of the type

$$P_{r_1\cdots r_s} = C_{r_1}\cdots C_{r_s}, \tag{41}$$

with $2(r_1 + r_2 + \cdots + r_s) = d$. Here

$$C_r = \mathrm{Tr}(\mathbf{F}^r), \tag{42}$$

defines the r^{th} Chern character of $SO(N)$. Now, since the curvature two-form $\mathbf{F}$ is in the vector representation it is antisymmetric in the group indices. Thus, the powers r_j in (42) are necessarily even, and therefore (41) vanishes unless d is a multiple of four. These results can be summarized in the following lemmas:

Lemma 1: For $d = 4k$, the only parity-odd d-forms built from e^a, R^{ab} and T^a, invariant under the AdS group, are the Chern characters for $SO(d+1)$.

Lemma 2: For $d = 4k + 2$, there are no parity-odd $SO(d+1)$-invariant d-forms constructed from e^a, R^{ab} and T^a.

In view of this, it is clear why attempts to construct purely gravitational theories with local AdS invariance in even dimensions have proven unsuccessful in spite of several serious efforts [19,30].

Since the expressions $P_{r_1\cdots r_s}$ in (41) are $4k$-dimensional closed forms, they are at best boundary terms which do not contribute to the classical equations (although they would assign different phases to configurations with nontrivial torsion in the quantum theory). The forms $P_{r_1\cdots r_s}$ can be expressed locally as the exterior derivative of a $(4k-1)$-form,

$$P_{r_1\cdots r_s} = dL^{AdS}_{T\ 4k-1}(W). \tag{43}$$

This implies that for each collection $\{r_1,\cdots r_s\}$, $L^{AdS}_{T\ 4k-1}$ is a good Lagrangian for the AdS group ($SO(4k)$) in $4k-1$ dimensions[9]. In a given dimension, the most general Lagrangian of this sort is a linear combination of all possible $L^{AdS}_{T\ 4k-1}$'s. It can be directly checked that these Lagrangians necessarily involve torsion explicitly.

These results can be summarized in the following

[9] The analysis is insensitive to the signature.

Theorem: In odd-dimensional spacetimes, there are two families of first-order gravitational Lagrangians $L(e,\omega)$, invariant under local AdS transformations:

a: Euler-Chern-Simons forms $L^{AdS}_{G\ 2n-1}$, in $d = 2n-1$ [parity-even]. Their exterior derivatives are the Euler characters in $2n$ dimensions and do not involve torsion explicitly, and

b: Pontryagin-Chern-Simons forms $L^{AdS}_{T\ 4k-1}$, in $d = 4k-1$ [parity-odd]. Their exterior derivatives are Chern characters in $4k$ dimensions and involve torsion explicitly.

It must be stressed that locally AdS-invariant gravity theories exist only in odd dimensions. They are *genuine* gauge systems, whose action comes from topological invariants in $d+1$ dimensions. These topological invariants can be written as the trace of a homogeneous polynomial of degree n in the AdS curvature. Obviously, for dimensions $4k-1$ both **a-** and **b-**families exist.

A. Examples for d=2n

In $d = 4$, the only local Lorentz-invariant 4-forms constructed with the recipe just described are [12]:

$$\begin{aligned}
\mathcal{E}_4 &= \epsilon_{abcd}R^{ab}R^{cd} \\
L_{EH} &= \epsilon_{abcd}R^{ab}e^ce^d \\
L_\Lambda &= \epsilon_{abcd}e^ae^be^ce^d \\
C_2 &= R^a_{\ b}R^b_{\ a} \\
L_{T_1} &= R^{ab}e_ae_b \\
L_{T_2} &= T^aT_a.
\end{aligned}$$

The first three terms are even under parity and the rest are odd. Of these, $\mathcal{E}_4$ and C_2 are topological invariant densities (closed forms): the Euler character and the second Chern character for $SO(4)$, respectively. The remaining four terms define the most general gravity action in four dimensions,

$$I = \int_{M_4}\left[\alpha L_{EH} + \beta L_\Lambda + \gamma L_{T_1} + \rho L_{T_2}\right]. \tag{44}$$

The two first terms in the R.H.S. can be combined with $\mathcal{E}_4$ into the Born-Infeld form (25) which is locally invariant under $SO(d)$, but not under $SO(d+1)$. It can also be seen, that by choosing $\gamma = -\rho$, the last two terms are combined into a topological invariant density, the Nieh-Yan form [31]. This choice implies that the entire odd part of the action becomes a boundary term. Furthermore, C_2, L_{T_1} and L_{T_2} can be combined into the second Chern character of the AdS group,

$$R^a_{\ b}R^b_{\ a} + \frac{2}{l^2}(T^aT_a - R^{ab}e_ae_b) = \bar{R}^A_{\ B}\bar{R}^B_{\ A}. \tag{45}$$

The form (45) is the only AdS invariant constructed just with e^a, ω^{ab} and their exterior derivatives, and therefore there are no locally AdS-invariant gravity theories in four dimensions.

In view of Lemmas (1) and (2), the corresponding AdS-invariant functionals in higher dimensions can be written in terms of the AdS curvature as linear combinations of terms like

$$\tilde{I}_{r_1\cdots r_s} = \int_M C_{r_1}\cdots C_{r_s}, \tag{46}$$

where $C_r = Tr[(\bar{R}^A_{\ B})^r]$ is the r-th Chern character for the AdS group, and $dim(M) = r_1 + \cdots + r_s$ is a multiple of four. For example, in $d = 8$ the Chern characters are

$$C_4 = Tr[(\bar{R}^A_{\ B})^4], \tag{47}$$

and

$$(C_2)^2 = Tr[(\bar{R}^A_{\ B})^2]_\wedge Tr[(\bar{R}^A_{\ B})^2]. \tag{48}$$

The corresponding integrals $\tilde{I}_4$ and $\tilde{I}_{2,2}$ are topological invariants that characterize the maps $SO(9) \to M_8$. Furthermore, as already mentioned, $\tilde{I}_{r_1\cdots r_s}$ vanishes if one of the r's happens to be odd, which is the case in $4k+2$ dimensions. Thus, one concludes that there are no torsional AdS-invariant gauge theories for gravity in even dimensions.

B. Examples for d=2n-1

The simplest example occurs in three dimensions, where there are two locally AdS invariant Lagrangians, namely, the Einstein-Hilbert with cosmological constant,

$$L^{AdS}_{G\ 3} = \frac{1}{l}\epsilon_{abc}[R^{ab}e^c + \frac{1}{3l^2}e^a e^b e^c], \tag{49}$$

and the "exotic" Lagrangian [15]

$$L^{AdS}_{T\ 3} = L^*_3(\omega) + 2e_a T^a, \tag{50}$$

where

$$L^*_3(\omega) \equiv \omega^a_{\ b} d\omega^b_{\ a} + \frac{2}{3}\omega^a_{\ b}\omega^b_{\ c}\omega^c_{\ a}. \tag{51}$$

The Lagrangians (49, 50, 51) are the Euler, the Pontryagin and the Lorentz Chern-Simons forms, respectively. Note that in (50), the local AdS symmetry fixes the relative coefficient between $L^*_3(\omega)$, and the torsion term $e_a T^a$. The most general action for gravitation in $d = 3$, which is invariant under $SO(4)$ is therefore the linear combination $\alpha L^{AdS}_{G\ 3} + \beta L^{AdS}_{T\ 3}$.

For $d = 4k-1$, the number of possible exotic forms grows as the partitions of k, $\mathsf{p}(k)$, in correspondence with the number of composite Chern invariants of the form:

$$P_{\{r_j\}} = \prod_{r_j\in \mathsf{p}(k)} C_{r_j}. \tag{52}$$

Thus, the most general Lagrangian in $4k-1$ dimensions takes the form

$$\kappa L^{AdS}_{G\ 4k-1} + \beta_{\{r_j\}} L^{AdS}_{T\ \{r_j\}\ 4k-1}, \tag{53}$$

where $dL^{AdS}_{T\ \{r_j\}\ 4k-1} = P_{r_1\cdots r_s}$, with $\sum_j r_j = 4k$. These Lagrangians are not boundary terms and, unlike the even-dimensional case, they have proper dynamics. For example, in seven dimensions one finds [29]

$$\begin{aligned} L^{AdS}_{T\ 7} &= \beta_{2,2}[R^a_{\ b}R^b_{\ a} + 2(T^a T_a - R^{ab}e_a e_b)]L^{AdS}_{T\ 3} \\ &+\beta_4[L^*_7(\omega) + 2(T^a T_a + R^{ab}e_a e_b)T^a e_a + 4T_a R^a_{\ b}R^b_{\ c}e^c], \end{aligned}$$

where L^*_{2n-1} is the Lorentz-CS $(2n-1)$-form,

$$dL^*_{2n-1}(\omega) = Tr[(R^a_{\ b})^n]. \tag{54}$$

It should be noted that the coefficients κ and $\beta_{\{r_j\}}$ are arbitrary and dimensionless. As is shown in the next section, these coefficients must be quantized by an extension of the argument used to prove that κ in (33) is also quantized [16].

C. Quantization of parameters

As it was shown in [16], in odd dimensions, the coefficient κ is quantized. We now extend that argument to show that the β's in (53) are also quantized. Consider the action for the connection W on a $(2n-1)$-dimensional, compact, oriented, simply connected manifold M without boundary, which is the boundary of an oriented $(2n)$-dimensional manifold Ω. By Stokes' theorem, the action for (53) can be written as an integral on Ω,

$$I^{AdS}_\Omega[W] = \int_\Omega \left(\bar{\kappa}\mathcal{E}_{2n} + \beta_{\{r\}}P_{\{r\}}\right). \tag{55}$$

(For $d = 4k+1$ the last term is absent as the $P_{\{r\}}$'s vanish). $I^{AdS}_\Omega[W]$ describes a topological field theory on Ω for W which should be insensitive to the change of Ω by another orientable manifold Ω' with the same boundary, i.e., $\partial\Omega = M = \partial\Omega$'. Thus we have

$$I^{AdS}_\Omega[W] = I^{AdS}_{\Omega'}[W] + \int_{\Omega\cup\Omega'} \left(\bar{\kappa}\mathcal{E}_{2n} + \beta_{\{r\}}P_{\{r\}}\right), \tag{56}$$

where the orientation of Ω' has been reversed. Now, $\Gamma := \Omega\cup\Omega'$ is a closed oriented manifold formed by joining Ω and Ω' continuously along M. Then (56) can be written as

$$I^{AdS}_\Omega[W] = I^{AdS}_{\Omega'}[W] + \bar{\kappa}\chi[\Gamma] + \beta_{\{r_j\}}p_{\{r_j\}}[\Gamma], \tag{57}$$

where $p_{\{r\}} = \int_\Gamma P_{\{r_j\}}$.

Substituting I_Ω by $I_{\Omega'}$ would have no effect in the path integral for the quantum theory provided the difference $\Delta[\Gamma] = I_\Omega - I_{\Omega'}$ is an integer multiple of Planck's constant h which, in addition, cannot change under continuous deformations of the fields. Thus, we have

$$\begin{aligned} \Delta[\Gamma] &= \bar{\kappa}\chi[\Gamma] + \beta_{\{r_j\}} c_{\{r_j\}}[\Gamma] \\ &= mh. \end{aligned} \tag{58}$$

Now, since the Euler and the Pontryagin numbers $\chi[\Gamma]$ and $p_{\{r_j\}}[\Gamma]$ are integers, the coefficients $\bar{\kappa}$ and $\beta_{\{r_j\}}$ are necessarily quantized.

The preceding argument is rigorously valid for a manifold with Euclidean signature. If M is locally Minkowskian one can apply the same reasoning to the analytic continuation of the path integral in which the base manifold M and its tangent space $T(M)_x$ are simultaneously Wick-rotated. This has the effect that the group of rotations on $T(M)_x$ may have nontrivial homotopy group, $\pi_{2n-1}[SO(2n)]$ so that the Chern characters can be nonzero.

IV. EXACT SOLUTIONS

As stressed above, the local symmetry of odd-dimensional gravity can be extended from Lorentz to AdS by an appropriate choice of the free coefficients in the action. The resulting Lagrangians –both with and without torsion terms– are Chern-Simons d-forms defined in terms of the AdS connection A, whose components include the vielbein and the spin connection [see equation (28)]. Thus, the field equations obtained by varying the vielbein and the spin connection respectively, can be written in an explicitly AdS-covariant form

$$< F^{n-1} J_{AB} >= 0, \tag{59}$$

where F= $\frac{1}{2}\bar{R}^{AB}J_{AB}$ is the AdS curvature with $\bar{R}^{AB}$ given by (29) and J_{AB} are the AdS generators.

Obviously, any locally AdS spacetime is a solution of (59). Apart from anti-de Sitter space itself, some interesting spacetimes with this feature are the topological black holes of Ref. [32], and some black branes with constant curvature worldsheet [33]. Also, for each d there is a unique static, spherically symmetric, asymptotically AdS black hole solution [21], as well as their topological extensions which have nontrivial event horizons [34]. Similarly, Friedman-Robertson-Walker cosmologies have also been found[10] [35].

[10] It can be seen that all the geometries described above can be extended into solutions of the gravitational BI theory in even dimensions.

A. Consequences of Local AdS Symmetry

The presence of a local AdS symmetry gives rise to an apparent paradox: Under the action of an AdS transformation which is not contained in the Lorentz subgroup (31), the curvature and torsion tensors transform as:

$$\begin{aligned} \delta\bar{R}^{ab} &= \frac{1}{l^2}(\lambda^a T^b - \lambda^b T^a), \\ \delta T^a &= -\bar{R}^a_{\ b}\lambda^b. \end{aligned} \tag{60}$$

Thus, in general, a solution with non-vanishing AdS curvature is mapped into another which is diffeomorphically inequivalent. In fact, the metric transform under (31) as

$$\delta g_{\mu\nu} = \delta_\xi g_{\mu\nu} - \xi^\lambda e_{a(\nu} T^a_{\mu)\lambda}, \tag{61}$$

where δ_ξ stands for a diffeomorphism whose parameter satisfy $\lambda^a = e^a_\mu \xi^\mu$. This implies that in the presence of torsion, the new metric is in general not "equivalent" to the old one. Furthermore, even if there is no torsion to begin with, by virtue of (61) the new metric will eventually be diffeomorphically inequivalent to higher order.

At first glance, it would seem that these two solutions would be physically inequivalent; in fact, these solutions have different geodesic structure. The apparent paradox stems from the fact that the geodesic equation is Lorentz covariant, but not AdS covariant. The crucial point, is what one means by "physically equivalent". The situation is analogous to the transformation of the electromagnetic field under a Lorentz boost: $\overrightarrow{E}$ and $\overrightarrow{B}$ fields transform, and even if one start with a purely magnetic (electric) configuration, to second order in (v/c) one finds both.

V. SUMMARY AND DISCUSSION

The presence of unconstrained torsion in higher dimensional gravity allows fixing the $[(d-1)/2]$ free parameters of the LL theory in terms of the gravitational and the cosmological constants. In even dimensions, it can be assumed the torsion to be a null vector of $\mathcal{T}_{ab}$ defined in (23), which is in general much weaker than imposing $T^a = 0$. The resulting theory has a Born-Infeld form, and possesses interesting exact solutions.

In odd dimensions, torsion needs not be constrained at all in the theory. This allows combining the vielbein and the spin connection as different components of an (A)dS or Poincaré connection. In this case the Lagrangian is a Chern-Simons form whose local symmetry is enlarged form $SO(d)$ to $SO(d+1)$ or $ISO(d)$.

The existence of propagating degrees of freedom –associated with the spacetime contorsion k^{ab}–, makes it natural to consider torsional terms explicitly in the Lagrangian as well. Such terms can be consistently added to the Lagrangian only $d = 4k-1$, which must possess local invariance under the (A)dS group ($SO(d+1)$). In the latter case, the Lagrangians are defined such that their exterior derivatives are given by all the possible products of the Chern characters in $4k$ dimensions, which are parity-odd. Thus, for $d = 4k-1$, the most general theory which allow the existence of independent propagating degrees of freedom for the contorsion, has a new set of parameters –the $\beta_{\{r\}}$'s–, which, together with the gravitational constant (κ), are necessarily quantized.

Chern-Simons theories are constructed on the basis of a single connection field on a fiber bundle. Contact with gravity is made by identifying one of the components of the gauge connection (e^a_μ) as the vielbein, that is, the soldering between the the base manifold (spacetime) and the tangent space. As shown in [15], in $2+1$ dimensions, one of the key points in quantizing gravity is to consider the fluctuations around vanishing vielbein and spin connection. That means that at the quantum level, spacetime metric is as relevant as in Yang-Mills theory. On the other hand, in the –purely metric– second order theory, writing the action requires the inverse metric, $g^{\mu\nu}$, which makes it impossible to work in the "unbroken phase" $e^a = \omega^{ab} = 0$).

The arguments traced above, suggest that the "extra ingredient" (identifying e^a_μ as the soldering) should be implemented at the last stage, at the classical level. On the other hand, CS actions have no dimensionful constants when written in terms of the connection W^{AB} (28), so that the fields have canonical dimension 1 and the action describes a bona fide AdS gauge system. The corresponding quantum theory as well as its local supersymmetric extensions would be renormalizable by power counting and possibly finite [16].

These theories, including torsional terms or not, possess a large class of interesting solutions, including variety of black holes for which the mass can be obtained through a surface integral [37] and homogeneous cosmologies[11].

The stability and positivity of the energy for the solutions of these theories is a nontrivial problem. However some insights can be gained from the supersymmetric extension of the theory, for in that case the expectation values of different charges are related and Bogomolny'i bounds can be established. The supersymmetric extension of gravity theories described here for $d = 4k-1$ was discussed in [10], and in general for $d = 2n-1$, in [11]. The key steps in that construction are: (a) Assume that pure gravity is described by the Chern-Simons Lagrangians including torsional terms from the start. (b) Identify the supersymmetric extension of AdS in odd dimensions as the gauge group, so that supersymmetry is realized in the fiber rather than in the base manifold[12]. (c) Find invariant tensors which allows writing the CS action for the super AdS connection which contains the pure gravitational theory considered in (a).

The minimal supersymmetric extensions of the AdS algebra can be constructed in a straightforward manner and with an explicit representation. The original scheme, devised by van Holten and Van Proeyen [38], gives the superalgebras in dimensions $1, 2, 3,\ mod 8$, but it can be easily generalized to all odd dimensions [29]. The first example of a supergravity action containing the LL-action was worked out by Chamseddine in five dimensions [20]. This construction, however cannot be generalized to arbitrary higher odd dimensions unless torsional terms are introduced in the gravitational sector. In $4k-1$ dimensions spinors in a chiral representation of $SO(4k)$ are required [39]. This can only be achieved if the gravitational Lagrangian is a particular combination of $L^{AdS}_{T\,2n-1}$ and $L^{AdS}_{G\,2n-1}$. This implies that minimal supergravity requires the inclusion of torsion.

In $d = 4k+1$ this issue does not arise because the torsion invariants in $d = 4k+2$ identically vanish, the corresponding supergravity theories are based only on L_G and require complex spinors and larger unitary superextensions of the AdS group as in [20].

Keeping in mind the above, it can shown that –around an appropriate background–, the conserved charges satisfy a central extension of the super AdS algebra[13]. Therefore, the Bogomolny'i bound on the bosonic charges can be read from the anticommutator of the fermionic symmetry generators, and as usual, solutions with Killing spinors saturate the bound. In $d = 2n-1$, one should expect the existence of a new kind of $2n$-dimensional theory at the boundary. That theory should be constructed on the generalization of the centrally extended gauge algebra, which can be viewed as the superconformal algebra at the boundary[14]. These theories should be a rich arena to test the recently conjectured AdS/CFT correspondence [41].

Acknowledgments

The authors are grateful to R. Aros, M. Contreras, J. Crisóstomo, J. Gamboa, C. Martínez, F. Méndez, R. Olea, M. Plyushchay, J. Saavedra, and C. Teitelboim for

[11]The problem of constructing conserved charges in Chern-Simons theories has been analyzed in [26] from a Hamiltonian point of view, and in [36], in the lagrangian approach. However, these approaches are inappropriate, for example, to compute the mass of solutions of a purely gravitational CS theory.

[12]An exceptionally simple case occurs when the coefficients α_p in the theory are chosen so that the bosonic system is locally Poincaré invariant. The supersymmetric extension was constructed in Ref. [22].

[13]The five-dimensional case was analyzed partially in [40].

[14]We thank Marc Henneaux for fruitful discussions about this point.

many enlightening discussions. We are specially thankful to M. Henneaux for many helpful comments and for his kind hospitality in Brussels. J.Z. also wishes to thank The Abdus Salam ICTP and the organizers of the "Extended Workshop in String Theory" for hospitaliy while writing up this work. This work was supported in part through grants 1990189, 1970151, 1980788, and 3960009 from FONDECYT, and grant 27-953/ZI-DICYT (USACH). The institutional support of Fuerza Aérea de Chile, I. Municipalidad de Las Condes, I. Municipalidad de Santiago, and a group of Chilean companies (AFP Provida, Business Design Associates, CGE, CODELCO, COPEC, Empresas CMPC, GENER S.A., Minera Escondida, NOVAGAS and XEROX-Chile) is also recognized.

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